Confluence Detection

Wieger Wesselink

September 2019

1 Confluence

Consider the following untimed linear process specification P, with initial state d_0.

$$P(d) = \sum_{i \in I} \sum_{e_i} c_i(d, e_i) \to a_i(f_i(d, e_i)) \cdot P(g_i(d, e_i))$$

We distinguish different kinds of confluence. Let summand $j \in I$ be the index of a τ-summand, and $i \in I$ be the index of an arbitrary summand. In the sequel we abbreviate $c_i(d, e_i)$ $f_i(d, e_i)$ and $g_i(d, e_i)$ with c_i, f_i and g_i. Note that for the τ-summand with index j we have $a_j = \tau$ and $f_j = []$.

1.1 Trivial confluence

Trivial confluence is defined as

$$C_{trivial}(i, j) = \forall d, e_i, e_j : (c_i \wedge c_j) \Rightarrow (a_i = \tau) \wedge (g_i = g_j)$$

Note that trivial confluence only applies to τ-summands. In isolation it is not a very useful property to check.

1.2 Triangular confluence

Triangular confluence is defined as

$$C_{triangular}(i, j) = \forall d, e_i, e_j : (c_i \wedge c_j) \Rightarrow (c_i[d := g_j] \wedge (f_i = f_i[d := g_j]) \wedge (g_i[d := g_j] = g_i))$$

1.3 Commutative confluence

Commutative confluence is defined as

$$\begin{array}{ll} C_{commutative}(i, j) = C_{trivial}(i, j) \vee \forall d, e_i, e_j : (c_i \wedge c_j) \Rightarrow & \exists e_i', e_j' : \\ \quad (& \\ \qquad\quad c_i[d := g_j, e_i := e_i')] & \\ \qquad \wedge \quad c_j[d := g_i, e_j := e_j'] & \\ \qquad \wedge \quad (f_i = f_i[d := g_j, e_i := e_i']) & \\ \qquad \wedge \quad (g_i[d := g_j, e_i := e_i'] = g_j[d := g_i, e_j := e_j']) & \\ \quad) & \end{array} \tag{1}$$

The reason for adding the term $C_{trivial}(i, j)$ is probably that otherwise a simple τ-summand like

$$(n = 0) \to \tau \cdot P(n = 1)$$

is not even confluent with itself.

1.4 Square confluence

Square confluence is defined as

$$C_{square}(i,j) = C_{trivial}(i,j) \vee \forall d, e_i, e_j : (c_i \wedge c_j) \Rightarrow c_i[d := g_j] \wedge c_j[d := g_i] \wedge (f_i = f_i[d := g_j]) \wedge (g_i[d := g_j] = g_j[d := g_i]))$$

It is obtained from $C_{commutative}(i,j)$ by taking $e'_i = e_i$ and $e'_j = e_j$.

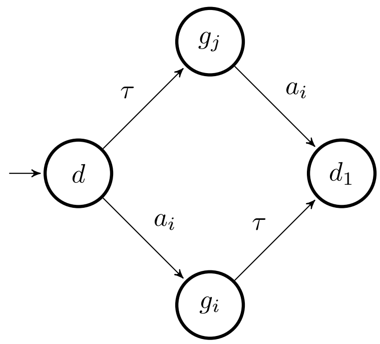

Figure 1: square commutative confluence, with $d_1 = g_j[d := g_i]$

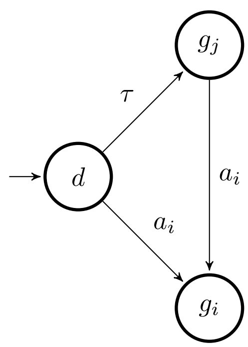

Figure 2: triangular confluence

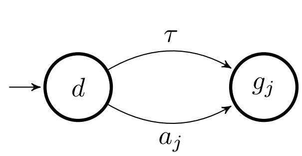

Figure 3: trivial confluence, with $a_j = \tau$